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We have developed algorithms which evolve in continuous time and which are capable of accomplishing a wide variety of computations of the type which can be used in making systems adaptive. Examples range from computations done by finite automata to differential equations which compute the eigenvalues of matrices.

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Summary

Although many biological and man-made systems combine aspects of digital and analog processing, until recently there has been very little theoretical work on models this type and many basic questions remain unresolved. In [1] we describe a family of input-output systems governed by ordinary differential equations i) whose behavior is robust in the sense that certain well-defined qualitative aspects of the output depend only on certain well-defined qualitative aspects of the input and ii) are capable of generating behavior of the type one usually associates with digital systems. It is show that rather simple differential equation models can robustly execute arithmetical and logical operations; in particular, we show that continuous-time dynamical systems can simulate arbitrary finite automata.

In reference [2] we establish a number of properties associated with the dynamical system $\dot{H} = [H,[H,N]]$, where H and N are symmetric n by n matrices and $[A,B] = AB - BA$. The most important of these come from the fact that this equation is equivalent to a certain gradient flow on the space of orthogonal matrices. We are especially interested in the role of this equation as an analog computer. For example, we show how to map the data associated with a linear programming problem into $H(0)$ and N in such a way as to have $\dot{H} = [H[H,N]]$ evolve to a solution of the linear programming problem. This result can be applied to find systems which solve a variety of generic combinatorial optimization problems and it even provides an algorithm for diagonalizing symmetric matrices and sorting lists of real numbers.

In a number of cases, combinatorial optimization problems can be recast as function minimization problems with the functions to be minimized taking the form

$$\eta(\Theta) = \sum_{i=1}^{n} \operatorname{tr} \Theta^T Q_i \Theta N_i - 2 \operatorname{tr} M \Theta^T$$

with Θ restricted to belong to a matrix Lie group. Not only is this of interest in itself, but because we are able to show that the method of steepest descent can be an effective tool for minimizing such functions, it immediately yields new computational procedures. Matching problems of the type studied here play an important role in computer vision, pattern analysis, and some types of learning theories. Often they are thought of as having both a continuous (geometrical) aspect involving the selection of a best fitting transformation and a combinatorial aspect involving a search over all possible permutations of some finite set. Karmarkar's work on linear programming already suggested that the difference between continuous and combinatorial optimization may not be all that profound; our work reinforces this point of view while considerably extending its scope. Reference [3] is devoted to this topic.

Publications

[1] R. W. Brockett, "Smooth Dynamical Systems Which Realize Arithmetic and Logical Operations," in *Three Decades of Mathematical System Theory. A Collection of Surveys at the Occasion of the Fiftieth Birthday of Jan C. Willems.* Lecture Notes in Control and Information Sciences Vol. 135 (H. Nijmeijer and J. M. Schumacher, eds.) Springer Verlag, Berlin, 1989, pp. 19-30.

[2] R. W. Brockett, "Dynamical Systems That Sort Lists, Diagonalize Matrices and Solve Linear Programming Problems," *Proceedings of of the 1988 IEEE Conference on Decision and Control*, December 1988.

[3] R. W. Brockett, "Least Squares Matching Problems," *Linear Algebra and Its Applications*, 1989. (to appear)